

# Collatz Space and 4N+1 Series Space

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## 1. Introduction

When Collatz problem<sup>1)</sup> is true, his rule arranges all odd numbers in a space. This space generates to 4N+1 series showed from the 2<sup>m</sup> infinite series (Table 1).

**Table 1. 4N+1 series and all odd numbers.**

$"A_1, A_2, A_3, A_4, A_5, \dots, A_{n-1}, A_n = 4*A_{n-1}+1, \dots"$							
(1), 5,	<b>21,</b>	85,	341,	1365,	$\dots$	$^{21}A_{n-1},$	$^{21}A_n = 4*^{21}A_{n-1}+1, \dots$
$x \equiv 2$	0	1	2	0	$\pmod{3}$		
<b>3,</b>	13,	53,	213,	853,	$\dots$	$^3A_{n-1},$	$^3A_n = 4*^3A_{n-1}+1, \dots$
$x \equiv 0$	1	2	0	1	$\pmod{3}$		
7,	29,	<b>117,</b>	469,	1877,	$\dots$	$^{117}A_{n-1},$	$^{117}A_n = 4*^{117}A_{n-1}+1, \dots$
$x \equiv 1$	2	0	1	2	$\pmod{3}$		
<b>9.</b>	37,	149,	597,	2389,	$\dots$	$^9A_{n-1},$	$^9A_n = 4*^9A_{n-1}+1, \dots$
$x \equiv 0$	1	2	0	1	$\pmod{3}$		
11,	<b>45,</b>	181,	725,	2901,	$\dots$	$^{45}A_{n-1},$	$^{45}A_n = 4*^{45}A_{n-1}+1, \dots$
$x \equiv 2$	0	1	2	0	$\pmod{3}$		
<b>15,</b>	61,	245,	981,	3925,	$\dots$	$^{15}A_{n-1},$	$^{15}A_n = 4*^{15}A_{n-1}+1, \dots$
$x \equiv 0$	1	2	0	1	$\pmod{3}$		
17,	<b>69,</b>	277,	1109,	4437,	$\dots$	$^{69}A_{n-1},$	$^{69}A_n = 4*^{69}A_{n-1}+1, \dots$
$x \equiv 2$	0	1	2	0	$\pmod{3}$		
19,	77,	<b>309,</b>	1237,	4949,	$\dots$	$^{309}A_{n-1},$	$^{309}A_n = 4*^{309}A_{n-1}+1, \dots$
$x \equiv 1$	2	0	1	2	$\pmod{3}$		
23,	<b>93,</b>	373,	1493,	5973,	$\dots$	$^{93}A_{n-1},$	$^{93}A_n = 4*^{93}A_{n-1}+1, \dots$
$x \equiv 2$	0	1	2	0	$\pmod{3}$		
.	.	.	.	.	.	.	.

All odd numbers can be arranged, using all 4N+1 series, in which  
 1st terms are 1, 3, 8u-1, 8u+1 and 8u+3 (u; natural numbers).  
 2nd, 3rd, 4th . . . terms in these series are always 8u+5.  
 In this report, we discuss the space arranging these 4N+1 series.

Each  $4N+1$  series stands in line “ $0, 1, 2, 0, 1, 2, 0, 1, 2, \dots \pmod{3}$ ” . Each triplet “ $8u-1, 8u+1$  and  $8u+3$ ” is “ $0, 1, 2 \pmod{3}$ ” . We can operate the unit of {  $A_p, A_{p+1}, A_{p+2}$  } ( $p$ ; natural number) in each series. In this report, we named each series the value of a term  $x \equiv 0 \pmod{3}$  in {  $A_1, A_2, A_3$  } .

Then, Collatz foresees “ $B_0 = 1$  at formula (1)” .

$$(3 * B_n + 1) / 2^m = B_{n-1} \quad \text{Formula (1).}$$

$B$  ; Every odd number,  $m, n$  ; Natural number.

The  $2^m$  series is standardized at 3 by this formula. We get formula (2), which is equivalent to formula (1).

$$B_n = (2^m * B_{n-1} - 1) / 3 \quad B_{n-1} ; x \equiv 1 \text{ or } 2 \pmod{3} \quad \text{Formula (2).}$$

When  $B_{n-1} = 1$ , we get main series(21-series) using  $2^m$  series ( $m$ ; even number).

$$\begin{array}{ccccccccc} 5, & 21, & 85, & 341, & 1365, & \dots, & {}^{21}A_{n-1}, & {}^{21}A_n = 4 * {}^{21}A_{n-1} + 1, & \dots \\ x \equiv 2 & 0 & 1 & 2 & 0 & & & & \end{array} \pmod{3}$$

”  ${}^{21}A_2 = 21$ ” is the billboard in {  ${}^{21}A_1, {}^{21}A_2, {}^{21}A_3$  } .  ${}^{21}A_1$  and  ${}^{21}A_3$  in {  ${}^{21}A_1, {}^{21}A_2, {}^{21}A_3$  } can be grafted by Formula (2).

## 2. Discussion I

We get 1st graft series (Figure 1.), grafting on main series.

Figure 1. 1st graft series on {  ${}^{21}A_1, {}^{21}A_2, {}^{21}A_3$  } .

$${}^{21}A_1 = 5 \quad x \equiv 2 \pmod{3}, \quad B_n = (2^m * 5 - 1) / 3, \quad (m; \text{ odd}).$$

$$\therefore \quad 3\text{-series}; \quad 3, \quad 13, \quad 53, \quad 213, \quad 853, \quad \dots, \quad {}^3A_{n-1}, \quad {}^3A_n = 4 * {}^3A_{n-1} + 1, \quad \dots$$

$${}^{21}A_2 = 21 \quad x \equiv 0 \pmod{3}, \quad \text{Non-graft.}$$

$${}^{21}A_3 = 85 \quad x \equiv 1 \pmod{3}, \quad B_n = (2^m * 85 - 1) / 3, \quad (m; \text{ even}).$$

$$\therefore \quad 453\text{-series}; \quad 113, \quad 453, \quad 1813, \quad \dots, \quad {}^{453}A_{n-1}, \quad {}^{453}A_n = 4 * {}^{453}A_{n-1} + 1, \quad \dots$$

And we get 2nd graft series, grafting on 1st Graft series. And then, we get 3rd graft

series, grafting on 2nd Graft series. . . . .

Then  $(n)$ -th graft series is gotten on  $(n-1)$ -th graft series.

We classify graft types into a, b and c, according to its shape. As each non-graft term is  $3*k$  ( $k$ ; odd), new graft series are expressed at Table 2.

Table 2. Graft types

$$a) \quad \{ {}^{3p}A_1, {}^{3p}A_2, {}^{3p}A_3 \} = \{ x \equiv 0, 1, 2 \pmod{3} \} \quad (p; \text{ odd number}) \quad \text{Formula(3).}$$

$${}^tA_1 = (4 * {}^{3p}A_2 - 1) / 3 = \{4 * (4 * {}^{3p}A_1 + 1) - 1\} / 3 = 16 * {}^{3p}A_1 / 3 + 1.$$

$${}^sA_1 = (2 * {}^{3p}A_3 - 1) / 3 = [2 * \{4 * (4 * {}^{3p}A_1 + 1) + 1\} - 1] / 3 = 2 * (16 * {}^{3p}A_1 / 3 + 1) + 1.$$

Where  ${}^sA_1$  is the center between  ${}^tA_1$  and  ${}^{3p}A_3$ .

$$b) \quad \{ {}^{3q}A_1, {}^{3q}A_2, {}^{3q}A_3 \} = \{ x \equiv 2, 0, 1 \pmod{3} \} \quad (q; \text{ odd number}) \quad \text{Formula(4).}$$

$${}^sA_1 = (2*{}^{3q}A_1 - 1)/3 = \{2*({}^{3q}A_2 - 1)/4 - 1\}/3 = ({}^{3q}A_2/3 - 1)/2.$$

$${}^t A_1 = (4 * {}^{3q} A_3 - 1) / 3 = \{4 * (4 * {}^{3q} A_2 + 1) - 1\} / 3 = 16 * {}^{3q} A_2 / 3 + 1.$$

$$c) \quad \{ {}^{3r}A_1, {}^{3r}A_2, {}^{3r}A_3 \} = \{ x \equiv 1, 2, 0 \pmod{3} \} \quad (r; \text{ odd number}) \quad \text{Formula(5).}$$

$${}^tA_1 = (4*{}^{3r}A_1 - 1)/3 = [4*\{{}^{3r}A_3 - 1\}/4 - 1]/3 = \{({}^{3r}A_3/3 - 1)/2 - 1\}/2.$$

$${}^sA_1 = (2*{}^{3r}A_2 - 1)/3 = \{2*({}^{3r}A_3 - 1)/4 - 1\}/3 = ({}^{3r}A_3/3 - 1)/2.$$

Where  ${}^sA_1$  is the center between  ${}^tA_1$  and  ${}^{3r}A_2$ .

when  ${}^tA_1, {}^sA_1$ ;  $x \equiv 0 \pmod{3}$ , we named each series the value of  $A_1$ .

when  $tA_1, sA_1$ ;  $x \equiv 1 \pmod{3}$ , we named each series the value of  $A_3 = 4*(4*A_1 + 1) + 1$ .

when  ${}^tA_1, {}^sA_1$ ;  $x \equiv 2 \pmod{3}$ , we named each series the value of  $A_2 = 4*A_1 + 1$ .

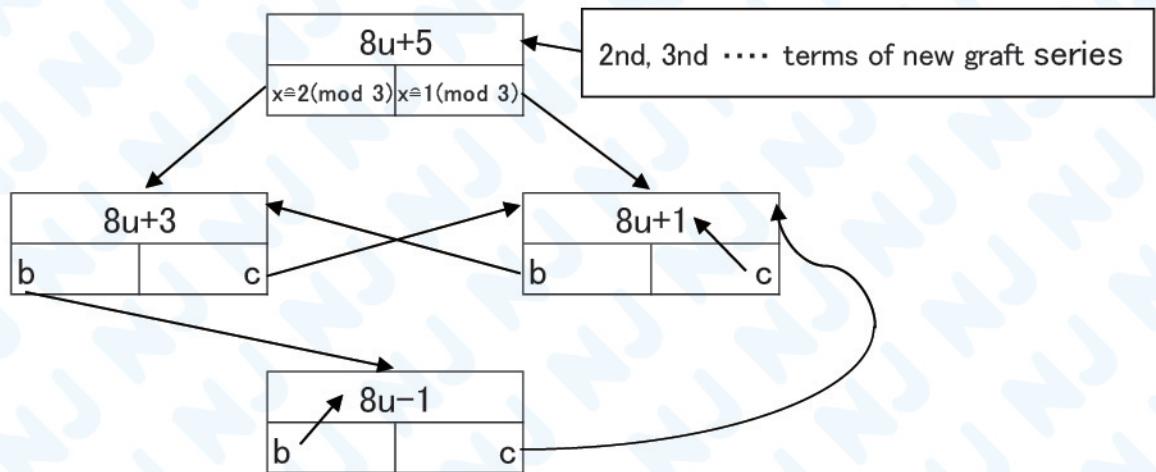
From Table 2, we have a constant stream of new  $3*k$ -series ( $k$ ; odd). Table 3 expresses many  $3*k$ -series from 1st-graft series to 40<sup>th</sup> graft series. These  $3*k$ -series increase by  $2^n$ . Therefore, Collatz space continues grafting infinitely for appearing all odd numbers.

**Table 3.** Graft structure of  $^{3k}An = 4 * ^{3k}An_{n-1} + 1$  ( $n$ ; natural number,  $k$ ; odd number) series.

### 3. Discussion II

We analyze 1st terms of these  $4N+1$  series grafting in Collatz space. Terms of main series are always  $8u+5$ .  $8u+3$  and  $8u+1$  graft on  $8u+5$ .  $8u-1$  graft on  $8u+3 \pmod{3}$  and are narrow path. We have examined the order of  $8u+5$ ,  $8u+3$ ,  $8u-1$  and  $8u+1$  (figure 2.).

Figure 2. When new graft series is b-type or c-type, next graft goes on.



“Each a-type” is the stop term(non-graft).

Where a, b & c of each term expresses  $x \equiv 0, 2 \& 1 \pmod{3}$  as same as Table 2.

We have arranged 1st terms of graft series in order of value. Its first page is Figure

3. At the beginning, the right  $8u+5$ ;  ${}^3A_2=13$  starts,  $\rightarrow 17 \rightarrow 11 \rightarrow 7 \rightarrow 9$  (a type) stops. Next, the left  $8u+5$ ;  ${}^{117}A_2=29$  starts,  $\rightarrow 19 \rightarrow 25 \rightarrow 33$  (a type) stops. . . . . .

Expressing new series, 2nd, 3rd , 4th. . . . . terms of new series are stocked to  $8u+5$ .

As we have observed till  ${}^3A_6=3413$  starts,  $\rightarrow 2275 \rightarrow 3033$  (a type) stops, 3~6 (including via  $8u+1$ ) terms of  $8u-1$  appeared sometimes continuously.

Figure 3. shows the important rule on graft path; a ( $8u+5$ ) starts  $\rightarrow$  some ( $8u+3$ ,  $8u-1$  or/and  $8u+1$ )  $\rightarrow$  a (a-type) stops. From the existence ratio of each term (Table 4),  $8u+5$  ( $x \equiv 1$  and  $2 \pmod{3}$ ) is  $2/12$ , and then a-type (stop term) of  $8u+3$ ,  $8u-1$  and  $8u+1$  is  $3/12$ .

$\therefore$  Collatz space does not have any part of a-type of  $8u+3$ ,  $8u-1$  and  $8u+1$ .

But Collatz space puts off these to infinite great and continues grafts. And more large numbers reach “1” by Formula (1). There is a fact; any part of a-type of  $8u+3$ ,  $8u-1$  and  $8u+1$  does not appear forever in Collatz space.

**Figure 3. The graft structure of  $4u+3$  and  $8u+1$ .**

$8u+5$		$4u+3 \pmod{3}$				$8u+1 \pmod{3}$		$8u+5$	
2 (mod 3); goto $8u+3$		goto $4u+3$		goto $8u+1$		goto $8u+3$		goto $8u+1$	
1	5 0.5 →	3	0.3 0						
		11	1.3 2	7	1	7	-1.1 1	9 0	9 1.1 0
7	29 3.5 →	19	2.3 1	25	1	15	-2.1 0	17 2.1 2	11 2 ← 13 1.5 3
		27	3.3 0			23	-3.1 2	15 0	25 3.1 1 33 0
3	13 6.5 →	35	4.3 2	23	2	31	-4.1 1	41 2	33 4.1 0
		43	5.3 1	57	0	39	-5.1 0		41 5.1 2 27 0
19	77 9.5 →	51	6.3 0			47	-6.1 2	31 1	49 6.1 1 65 2 ← 37 4.5 9
		59	7.3 2	39	0	55	-7.1 1	73 1	57 7.1 0
25	101 12.5 →	67	8.3 1	89	2	63	-8.1 0		65 8.1 2 43 1
		75	9.3 0			71	-9.1 2	47 2	73 9.1 1 97 1
31	125 15.5 →	83	10.3 2	55	1	79	-10.1 1	105 0	81 10.1 0 ← 61 7.5 15
		91	11.3 1	121	1	87	-11.1 0		89 11.1 2 59 2
9	37 149 18.5 →	99	12.3 0			95	-12.1 2	63 0	97 12.1 1 129 0
		107	13.3 2	71	2	103	-13.1 1	137 2	105 13.1 0
43	173 21.5 →	115	14.3 1	153	0	111	-14.1 0		113 14.1 2 75 0 ← 85 10.5 21 5
		123	15.3 0			119	-15.1 2	79 1	121 15.1 1 161 2
49	197 24.5 →	131	16.3 2	87	0	127	-16.1 1	169 1	129 16.1 0
		139	17.3 1	185	2	135	-17.1 0		137 17.1 2 91 1
55	221 27.5 →	147	18.3 0			143	-18.1 2	95 2	145 18.1 1 193 1 ← 109 13.5 27
		155	19.3 2	103	1	151	-19.1 1	201 0	153 19.1 0
15	61 245 30.5 →	163	20.3 1	217	1	159	-20.1 0		161 20.1 2 107 2
		171	21.3 0			167	-21.1 2	111 0	169 21.1 1 225 0
67	269 33.5 →	179	22.3 2	119	2	175	-22.1 1	233 2	177 22.1 0 ← 133 16.5 33
		187	23.3 1	249	0	183	-23.1 0		185 23.1 2 123 0
73	293 36.5 →	195	24.3 0			191	-24.1 2	127 1	193 24.1 1 257 2
		203	25.3 2	135	0	199	-25.1 1	265 1	201 25.1 0
79	317 39.5 →	211	26.3 1	281	2	207	-26.1 0		209 26.1 2 139 1 ← 157 19.5 39
		219	27.3 0			215	-27.1 2	143 2	217 27.1 1 289 1
21	85 341 42.5 →	227	28.3 2	151	1	223	-28.1 1	297 0	225 28.1 0
		235	29.3 1	313	1	231	-29.1 0		233 29.1 2 155 2
91	365 45.5 →	243	30.3 0			239	-30.1 2	159 0	241 30.1 1 321 0 ← 181 22.5 45 11
		251	31.3 2	167	2	247	-31.1 1	329 2	249 31.1 0
97	389 48.5 →	259	32.3 1	345	0	255	-32.1 0		257 32.1 2 171 0
		267	33.3 0			263	-33.1 2	175 1	265 33.1 1 353 2
103	413 51.5 →	275	34.3 2	183	0	271	-34.1 1	361 1	273 34.1 0 ← 205 25.5 51
		283	35.3 1	377	2	279	-35.1 0		281 35.1 2 187 1
27	109 437 54.5 →	291	36.3 0			287	-36.1 2	191 2	289 36.1 1 385 1
		299	37.3 2	199	1	295	-37.1 1	393 0	297 37.1 0
115	461 57.5 →	307	38.3 1	409	1	303	-38.1 0		305 38.1 2 203 2 ← 229 28.5 57
		315	39.3 0			311	-39.1 2	207 0	313 39.1 1 417 0
121	485 60.5 →	323	40.3 2	215	2	319	-40.1 1	425 2	321 40.1 0
		331	41.3 1	441	0	327	-41.1 0		329 41.1 2 219 0
127	509 63.5 →	339	42.3 0			335	-42.1 2	223 1	337 42.1 1 449 2 ← 253 31.5 63
		347	43.3 2	231	0	343	-43.1 1	457 1	345 43.1 0

**Table 4. The existence ratio of each term.**

	8u+5	8u+3	8u-1	8u+1
Terms: $x \equiv 0 \pmod{3}$	1/12	1/12	1/12	1/12
Terms: $x \equiv 2 \pmod{3}$	1/12	1/12	1/12	1/12
Terms: $x \equiv 1 \pmod{3}$	1/12	1/12	1/12	1/12

#### 4. Reference

1). Ishikura, Tetuya; Asahi newspaper 2021, 1026 (Tue), Kanagawa-Morning-Version.

“ Collatz Problem, 宇宙人が仕向けた罠.”

研究者 ; 上嶋信幸 ほか2名。

Kouza-Agri-Develop-Room, Exel-Group.

#### 5. Appendix

- Table 5. We see the projection of 4N+1 series, comparing Step up with First down.

Table 5. We see the projection of  $4N+1$  series ,comparing Step-up with First-down.